University of Antwerp
Operations Research Group
ANT/OR

A history of metaheuristics

Kenneth Sörensen Marc Sevaux Fred Glover

ORBEL29 – Antwerp – 5-6 February 2015

A history of this talk

Handbook of Heuristics (Springer)



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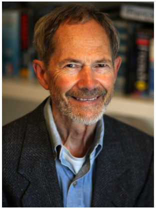

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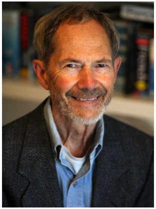


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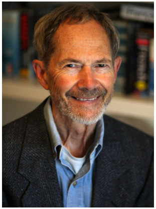

Methodology

- High-level perspective
- Not an annotated chronological bibliography
- Attempt to discover paradigm-shifts
- No futile attempts to adopt a neutral perspective

What is a heuristic?

$$x^* = \arg\max_{x \in X} f(x)$$

Exact method

Optimization method **with** guarantee of optimality

Heuristic

Optimization method **without** guarantee of optimality

What is a metaheuristic?

Metaheuristic ver. 1 (framework)

A metaheuristic is a *high-level*, *problem-independent* algorithmic *framework* that provides a set of guidelines or strategies to develop heuristic optimization algorithms.

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Metaheuristic ver. 2 (algorithm)

The term is also used to refer to a *problem-specific implementation* of a heuristic optimization algorithm according to the guidelines expressed in such a framework.

Five periods of (meta)heuristics

1.	The pre-theoretical period	(until c. 1940)
2.	The early period	(c. 1940 – c. 1980)
3.	The method-centric period	(c. 1980 – c. 2000)
4.	The framework-centric period	(c. 2000 – now)
5.	The scientific period	(the future)

The pre-theoretical period

- Optimization problems are all around us
- The human mind is naturally equipped with an incredibly versatile *heuristic* solver

The pre-theoretical period

- Optimization problems are all around us
- The human mind is naturally equipped with an incredibly versatile *heuristic* solver

Sörensen's conjecture

In the real world, solving optimization problems using exact methods is a waste of resources.

The pre-theoretical period

- Optimization problems are all around us
- The human mind is naturally equipped with an incredibly versatile *heuristic* solver
- It has meta-strategies ("meta-heuristics") too, e.g.,
  - learning by analogy
  - greediness
  - most difficult first
  - means-end-analysis ("local search")
  - don't do something that failed in the past ("tabu search")
  - ...

Sörensen's conjecture

In the real world, solving optimization problems using exact methods is a waste of resources.

The early period

- After WWII
- Coincides with development of OR
- “How to solve it” (1945)
  - “Analogy”
  - “Induction”
  - “Auxiliary problem”
- High-level algorithmic ideas
  1. Constructive heuristics
  2. Regret algorithms

George Pólya

The early period

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Vol. 6, No. 1, Jan.-Feb., 1958
Printed in U.S.A.

HEURISTIC PROBLEM SOLVING: THE NEXT ADVANCE IN OPERATIONS RESEARCH*

Herbert A. Simon and Allen Newell

Carnegie Institute of Technology, Pittsburgh, Pennsylvania, and *The Rand Corporation, Santa Monica, California*

THE IDEA THAT the development of science and its application to human affairs often requires the cooperation of many disciplines and professions will not surprise the members of this audience. Operations research and management science are young professions that are only now beginning to develop their own programs of training; and they have meanwhile drawn their practitioners from the whole spectrum of intellectual disciplines. We are mathematicians, physical scientists, biologists, statisticians, economists, and political scientists.

In some ways it is a very new idea to draw upon the techniques and fundamental knowledge of these fields in order to improve the everyday operation of administrative organizations. The terms 'operations research' and 'management science' have evolved in the past fifteen years, as have the organized activities associated with them. But of course, our professional activity, the application of intelligence in a systematic way to administration, has a history that extends much farther into the past. One of its obvious antecedents is the scientific management movement fathered by Frederick W. Taylor.

But for an appropriate patron saint for our profession, we can most appropriately look back a full half century before Taylor to the remarkable figure of Charles Babbage. Perhaps more than any man since Leonardo da Vinci he exemplified in his life and work the powerful ways in which

* Address at the banquet of the Twelfth National Meeting of the Operations Research Society of America, Pittsburgh, Pennsylvania, November 14, 1957. Mr. Simon presented the paper; its content is a joint product of the authors. In this, they rely on the precedent of Genesis 27:22, "The voice is Jacob's voice, but the hands are the hands of Esau."

- Artificial intelligence as the basis for heuristic design
- Realization that some ideas on the design of heuristics can be generalized

The method-centric period

- From the 60s: evolutionary methods
  - Evolution strategies (Schwefel, Rechenberg) – no population or crossover
  - Genetic algorithms (Holland, Goldberg): population + crossover
  - Theoretical studies to “prove” convergence
  - General sentiment: an all-powerful black-box optimizer within reach
- 1980s: another metaphor: simulated annealing
- 1980s: more AI-based methods
  - Local search
  - Threshold accepting
  - Tabu search
  - A few more

Meta-heuristics introduced

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FUTURE PATHS FOR INTEGER PROGRAMMING AND LINKS TO ARTIFICIAL INTELLIGENCE

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Center for Applied Artificial Intelligence, Graduate School of Business, University of Colorado, Boulder, CO 80309, U.S.A.

Scope and Purpose—A summary is provided of some of the recent (and a few not-so-recent) developments that offer promise for enhancing our ability to solve combinatorial optimization problems. These developments may be usefully viewed as a synthesis of the perspectives of operations research and artificial intelligence. Although compatible with the use of algorithmic subroutines, the frameworks examined are primarily heuristic, based on the supposition that effective solution of complex combinatorial structures in some cases may require a level of flexibility beyond that attainable by methods with formally demonstrable convergence properties.

Abstract—Integer programming has benefited from many innovations in models and methods. Some of the promising directions for elaborating these innovations in the future may be viewed from a framework that links the perspectives of artificial intelligence and operations research. To demonstrate this, four key areas are examined: (1) controlled randomization, (2) learning strategies, (3) induced decomposition and (4) tabu search. Each of these is shown to have characteristics that appear usefully relevant to developments on the horizon.

1. INTRODUCTION

Integer programming (IP) has gone through many phases in the last three decades, spurred by the recognition that its domain encompasses a wide range of important and challenging practical applications. Two of the more prominent landmarks in the development of the field have undoubtedly been the emergence of the cutting plane and branch and bound approaches. As general solution strategies, these approaches have drawn on concepts from diverse areas including number theory, group theory, logic, convex analysis, nonlinear functions, and matroid theory [1–7].

From the theoretical side, cutting planes have received the greatest attention, though from a broad perspective the distinction between cutting plane and branch and bound methods blurs. Indeed, branch and bound may be viewed as *provisional cutting*. From the practical side, the most effective general purpose methods have relied heavily on branch and bound, conceiving branch and bound in its standard (narrower) sense, where the collection of provisional cuts derives simply from constraining integer variables to satisfy lower and upper bounds. Doses of cutting plane theory have been used to improve the basic branch and bound framework, chiefly by generating cuts to be added before initiating the branch and bound process (or in some cases just prior to selecting a next branch) [8–14]. The cuts used, however, are typically those that are easily derived and generated. The more labyrinthine and esoteric derivations have not so far demonstrated great practical utility.

Implicit in cutting methods and branch and bound methods are the allied notions of problem

Discussion

"Meta" or "Modern" heuristics?

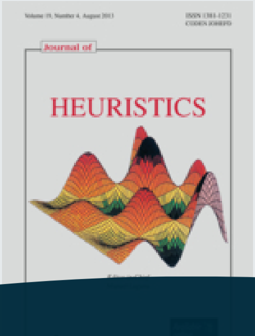
The method-centric period

- General sentiment: metaheuristics as recipes
- Neural networks
- New methods
  - GRASP
  - Ant colony optimization
- Second half of the 1990s: disappointment over reachability of über-powerful black-box optimizers
- No free lunch theorem

1995

Metaheuristics International Conference **MIC**

1995



The framework-centric period

- Introduction of *hybrid* metaheuristics (e.g., memetic algorithms)
- Mix-and-match of metaheuristic components
- Realization that metaheuristics should be seen as frameworks, rather than methods.
- **Math**euristics

2001

EU/ME

The metaphor-centric period

Swarm intelligence based algorithms			Bio-inspired (not SI-based) algorithms		
Algorithm	Author	Reference	Algorithm	Author	Reference
Accelerated PSO	Yang et al.	[69], [71]	Atmosphere clouds model	Yan and Hao	[67]
Ant colony optimization	Dorigo	[15]	Biogeography-based optimization	Simon	[56]
Artificial bee colony	Karaboga and Basturk	[31]	Brain Storm Optimization	Shi	[55]
Bacterial foraging	Passino	[46]	Differential evolution	Storn and Price	[57]
Bacterial-GA Foraging	Chen et al.	[6]	Dolphin echolocation	Kaveh and Farhoudi	[33]
Bat algorithm	Yang	[78]	Japanese tree frogs calling	Hernández and Blum	[28]
Bee colony optimization	Teodorović and Dell'Orco	[62]	Eco-inspired evolutionary algorithm	Parpinelli and Lopes	[45]
Bee system	Lucic and Teodorovic	[40]	Egyptian Vulture	Sur et al.	[59]
BeeHive	Wedde et al.	[65]	Fish-school Search	Lima et al.	[14], [3]
Wolf search	Tang et al.	[61]	Flower pollination algorithm	Yang	[72], [76]
Bees algorithms	Pham et al.	[47]	Gene expression	Ferreira	[19]
Bees swarm optimization	Drias et al.	[16]	Great salmon run	Mozaffari	[43]
Bumblebees	Comellas and Martinez	[12]	Group search optimizer	He et al.	[26]
Cat swarm	Chu et al.	[7]	Human-Inspired Algorithm	Zhang et al.	[80]
Consultant-guided search	Iordache	[29]	Invasive weed optimization	Mehrabian and Lucas	[42]
Cuckoo search	Yang and Deb	[74]	Marriage in honey bees	Abbass	[1]
Eagle strategy	Yang and Deb	[75]	OptBees	Maia et al.	[41]
Fast bacterial swarming algorithm	Chu et al.	[8]	Paddy Field Algorithm	Premaratne et al.	[48]
Firefly algorithm	Yang	[70]	Roach infestation algorithm	Havens	[25]
Fish swarm/school	Li et al.	[39]	Queen-bee evolution	Jung	[30]
Good lattice swarm optimization	Su et al.	[58]	Shuffled frog leaping algorithm	Eusuff and Lansey	[18]
Glowworm swarm optimization	Krishnanand and Ghose	[37], [38]	Termite colony optimization	Hedayatzadeh et al.	[27]
Hierarchical swarm model	Chen et al.	[5]	Physics and Chemistry based algorithms		
Krill Herd	Gandomi and Alavi	[22]	Big bang-big Crunch	Zandi et al.	[79]
Monkey search	Mucherino and Seref	[44]	Black hole	Hatamlou	[24]
Particle swarm algorithm	Kennedy and Eberhart	[35]	Central force optimization	Formato	[21]
Virtual ant algorithm	Yang	[77]	Charged system search	Kaveh and Talatahari	[34]
Virtual bees	Yang	[68]	Electro-magnetism optimization	Cuevas et al.	[13]
Weightless Swarm Algorithm	Ting et al.	[63]	Galaxy-based search algorithm	Shah-Hosseini	[53]
Other algorithms			Gravitational search	Rashedi et al.	[50]
Anarchic society optimization	Shayeghi and Dadashpour	[54]	Harmony search	Geem et al.	[23]
Artificial cooperative search	Civicioglu	[9]	Intelligent water drop	Shah-Hosseini	[52]
Backtracking optimization search	Civicioglu	[11]	River formation dynamics	Rabanal et al.	[49]
Differential search algorithm	Civicioglu	[10]	Self-propelled particles	Vicsek	[64]

The metaphor-centric period

A dark page in history

Swarm intelligence based algorithms

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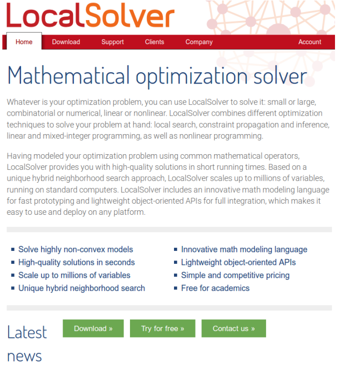
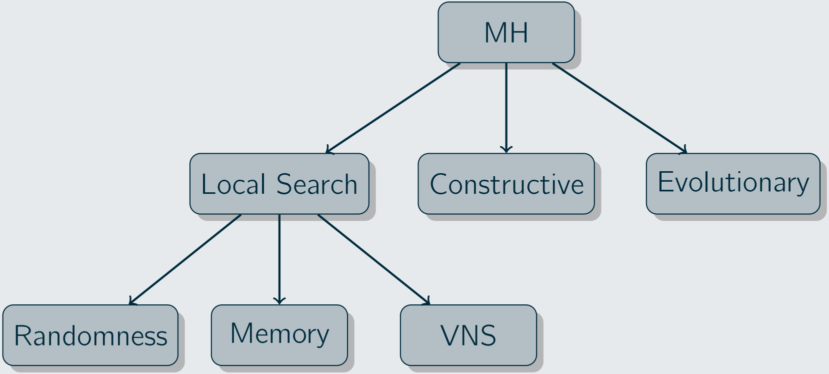
Where are we now?

- Metaheuristics have lived up to their promise: heavily used in real-life systems
- Widespread agreement that metaheuristics are not recipes
- Still: not a lot of "solvers", still largely an art



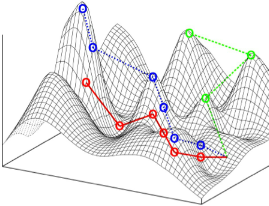
Where are we now?

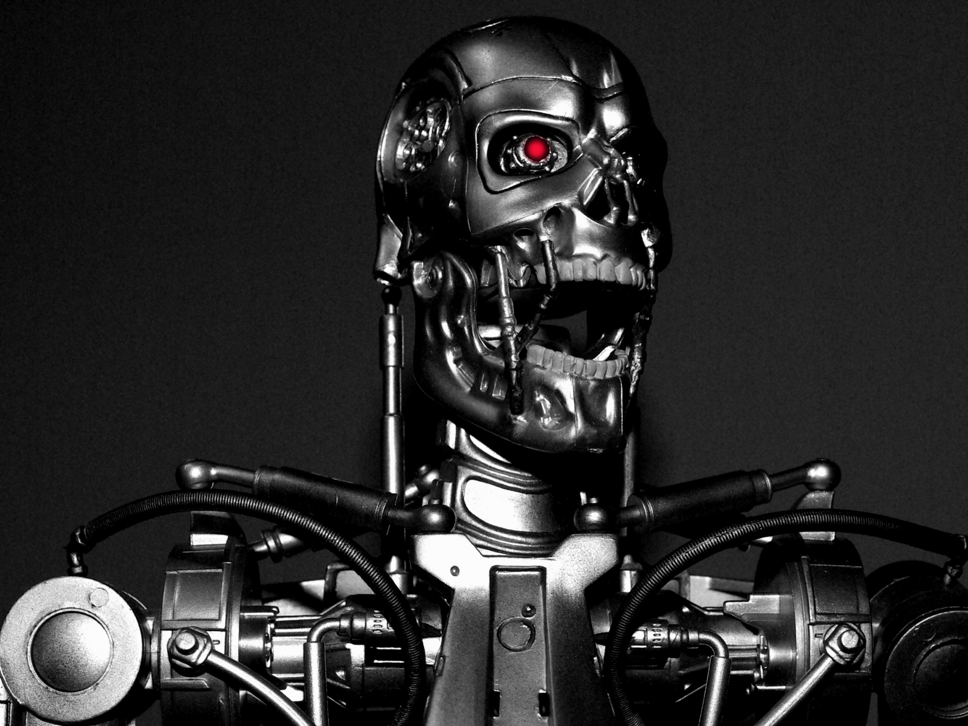
Taxonomy of metaheuristics creates clarity



The future: the scientific period

- Growing up of the field of metaheuristics as a *science*
  - Understanding the behavior of metaheuristics
  - Adequate testing protocols
  - Decomposition
  - Knowledge > performance
- Development of powerful solvers to decrease development time
- A more natural language to formulate optimization problems
- Availability of dedicated tools, including exact methods and constraint programming





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comex
combinatorial optimization:
metaheuristics & exact methods