Set-Based Modeling in LocalSolver 6.0

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20 years of practice and research
http://www.innovation24.fr

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LocalSolver

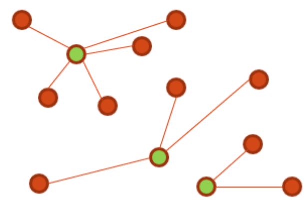
Hybrid math optimization solver

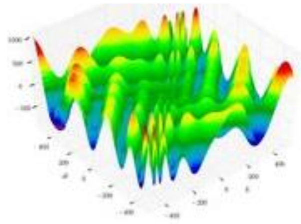
For combinatorial, numerical,
or mixed-variable optimization

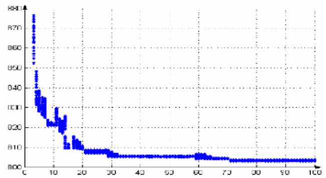
Suited for large-scale
non-convex optimization

Quality solutions in seconds
without tuning

LocalSolver
=
LS + CP/SAT + LP/MIP + NLP







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Motivations

Modeling approaches for

the *Traveling Salesman Problem*

Mixed-Integer Programming

With an **exponential number** of constraints

Minimise $\sum_{\substack{i,j \\ i \neq j}} c_{ij} x_{ij}$

Conventional Formulation (C) (Dantzig, Fulkerson and Johnson (1954))

$$\sum_{\substack{j \\ j \neq i}} x_{ij} = 1 \qquad \forall\ i \in N$$

$$\sum_{\substack{i \\ i \neq j}} x_{ij} = 1 \qquad \forall\ j \in N$$

$$\sum_{\substack{i,j \in M \\ i \neq j}} x_{ij} \leq |M| - 1 \qquad \forall\ M \subset N \text{ such that } \{1\} \notin M, |M| \geq 2$$

→ Iterative procedure to add subtour elimination constraints

Variant with *O(n²)* variables and constraints

SINGLE COMMODITY FLOW (**F1**) (Gavish and Graves (1978))

Both constraints are retained but we also introduce (continuous) variables:

y_{ij} = 'Flow' in an arc (i,j) $i \neq j$

and constraints:

$$y_{ij} \leq (n-1) x_{ij} \qquad \forall\ i,j \in N,\ i \neq j$$

$$\sum_{\substack{j \\ j \neq 1}} y_{1j} = n - 1$$

$$\sum_{\substack{i \\ i \neq j}} y_{ij} - \sum_{\substack{k \\ i \neq k}} y_{jk} = 1 \qquad \forall\ j \in N - \{1\}$$

In Orman & Williams : A survey of different integer programming formulations of the TSP

LocalSolver 5.0

A compact model based on positions

```
/* Declares the optimization model. */
function model() {
    // x[i][j] equal to 1 if city j is ith visited city in the tour
    x[0..nbCities-1][0..nbCities-1] <- bool();

    // one city per position i
    for [i in 0..nbCities-1] constraint sum[j in 0..nbCities-1](x[i][j]) == 1;
    // one position per city j
    for [j in 0..nbCities-1] constraint sum[i in 0..nbCities-1](x[i][j]) == 1;

    //city[i] is the city at position i in the tour
    city[i in 0..nbCities-1] <- sum[j in 0..nbCities-1](j*x[i][j]);

    // Distance of the arc reaching the ith city
    distance[0] <- distanceWeight[city[nbCities - 1]][city[0]];
    distance[i in 1..nbCities - 1] <- distanceWeight[city[i - 1]][city[i]];

    // Minimize the total distance
    obj <- sum[i in 0..nbCities - 1](distance[i]);

    minimize obj;
}
```

→ A polynomial-size model (not an iterative procedure)

→ No artificial variables and constraints

→ But still based on binary decisions

Natural Modeling

As a permutation

The Traveling Salesman Problem (TSP)

TSP: Given a list of cities and their pairwise distances, find a shortest possible tour that visits each city exactly once.

Objective: find a permutation $a_1,\ldots,a_n$ of the cities that minimizes

$$d(a_1, a_2) + d(a_2, a_3) + \ldots + d(a_{n-1}, a_n) + d(a_n, a_1)$$

where $d(i, j)$ is the distance between cities i and j

An optimal TSP tour through Germany's 15 largest cities

In Kenneth R. Rosen: Permutations and Combinations.

Reference modeling

Garey & Johnson

[ND22] **TRAVELING SALESMAN**

INSTANCE: Set C of m cities, distance $d(c_i, c_j) \in Z^+$ for each pair of cities $c_i, c_j \in C$, positive integer B.

QUESTION: Is there a tour of C having length B or less, i.e., a permutation $< c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} >$ of C such that

$$\left[\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)}) \right] + d(c_{\pi(m)}, c_{\pi(1)}) \leqslant B \quad ?$$

Set-based modeling

Innovative modeling concepts
for routing & scheduling problems

List Variables

Structured decisional operator list(n)

- Order a subset of values in domain {0, …, n-1}
- Each value is unique in the list

Classical operators to interact with “list”

- count(u): number of values selected in the list
- at(u,i) or u[i]: value at index i in the list
- indexOf(u,v): index of value v in the list
- contains(u,v): equivalent to “indexOf(u,v) != -1”
- disjoint(u1, u2, …, uk): true if u1, u2, …, uk are pairwise disjoint
- partition(u1, u2, …, uk): true if u1, u2, …, uk induce a partition of {0, …, n-1}

Traveling salesman

```
function model() {
  x <- list(N) ; // order n cities {0, ..., n-1} to visit
  constraint count(x) == N; // exactly n cities to visit
  minimize sum[i in 1..N-1]( Dist[ x[i-1] ][ x[i] ] )
                 + Dist[ x[N-1] ][ x[0] ] ; // minimize sum of traveled distances
}
```

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Objective: find a permutation $a_1,\ldots,a_n$ of the cities that minimizes

$$d(a_1, a_2) + d(a_2, a_3) + \ldots + d(a_{n-1}, a_n) + d(a_n, a_1)$$

where $d(i, j)$ is the distance between cities i and j

An optimal TSP tour through Germany's 15 largest cities

Why not a single line model ?

```
constraint TSP(graph);
```

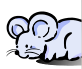
Real-world models do not fit in tight frameworks

$$\mathrm{Time}[A,B,T] = \frac{2({\alpha_x}^2+{\alpha_y}^2)}{-2(\alpha_x\beta_x+\alpha_y\beta_y)+\sqrt{4(\alpha_x\beta_x+\alpha_y\beta_y)^2-4({\alpha_x}^2+{\alpha_y}^2)({\beta_x}^2+{\beta_y}^2-V^2)}}+T$$

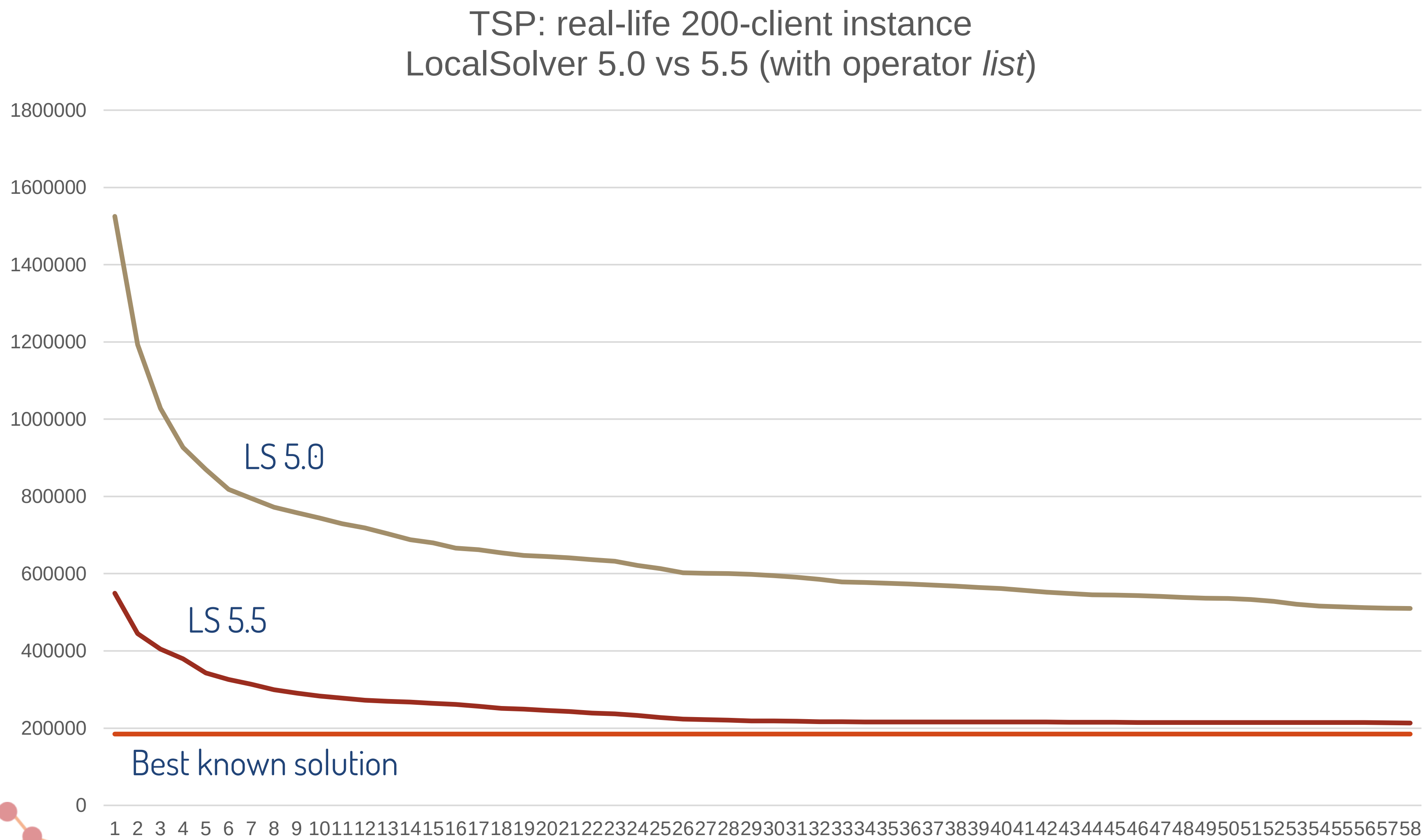
With $\alpha_x, \beta_x, \alpha_y, \beta_y$ function of A,B and T

```
constraint TSP(graph);
```
?

Kinetic TSP



Performance ?



Vehicle routing

```
function model() {
  x[1..K] <- list(N) ; // for each truck, order the clients to visit
  constraint partition( x[1..K] ); // each client is visited once
  distances[k in 1..K] <- sum[i in 1..N-1]( dist( x[k][i-1], x[k][i]) )
             + dist( x[k][N-1], x[k][0] ); // traveled distance for each truck

  minimize sum[k in 1..K]( distances[k] ); // minimize total traveled distance
}
```

	TSP	VRP
Normal	Count(x)=N	partition(x[1..K])
Prize-collecting	maximize sum(…)	disjoint(x[1..K])

CVRP benchmarks

CVRP - instances A

- 32 to 80 clients, 10 trucks max.
- 27 instances
- 5 minutes of running time
- LS binary: almost infeasible
- LS list: 1 % avg. opt. gap

CVRP - instances X100-500

- 100 to 500 clients, 138 trucks max.
- 67 instances
- 5 minutes of running time
- LS binary: almost infeasible
- LS list: 9 % avg. opt. gap

CVRPTW benchmarks

CVRPTW – instances Solomon R100

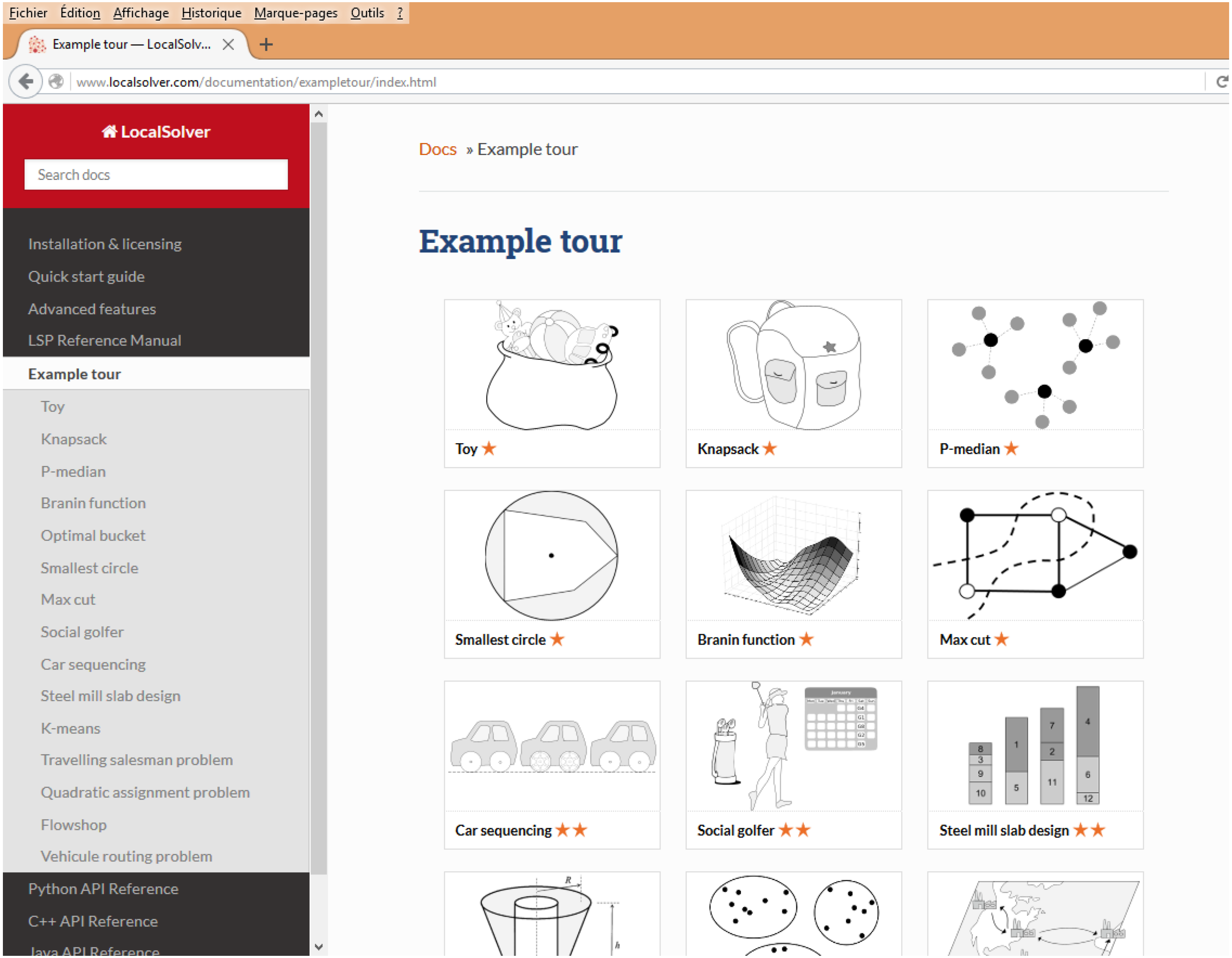
- 101 to 112 clients, 19 trucks max.
- 13 instances
- 5 minutes of running time
- LS binary: N/A
- LS list: 3 % avg. opt. gap

CVRPTW – instances Solomon R200

- 201 to 208 clients, 4 trucks max.
- 8 instances
- 5 minutes of running time
- LS binary: N/A
- LS list: 8 % avg. opt. gap

Beyond routing problems

Scheduling, planning, sequencing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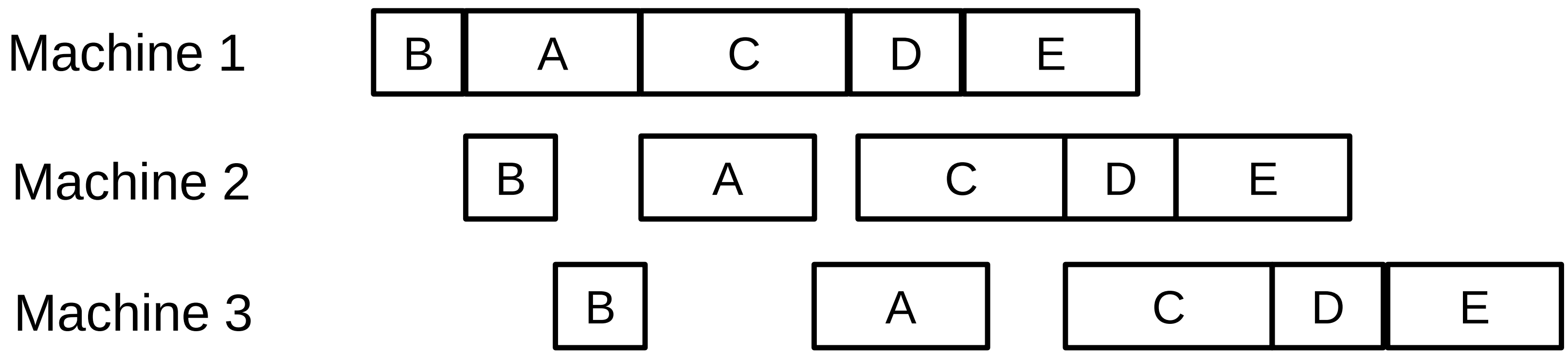
Social golfer ★★
Steel mill slab design ★★

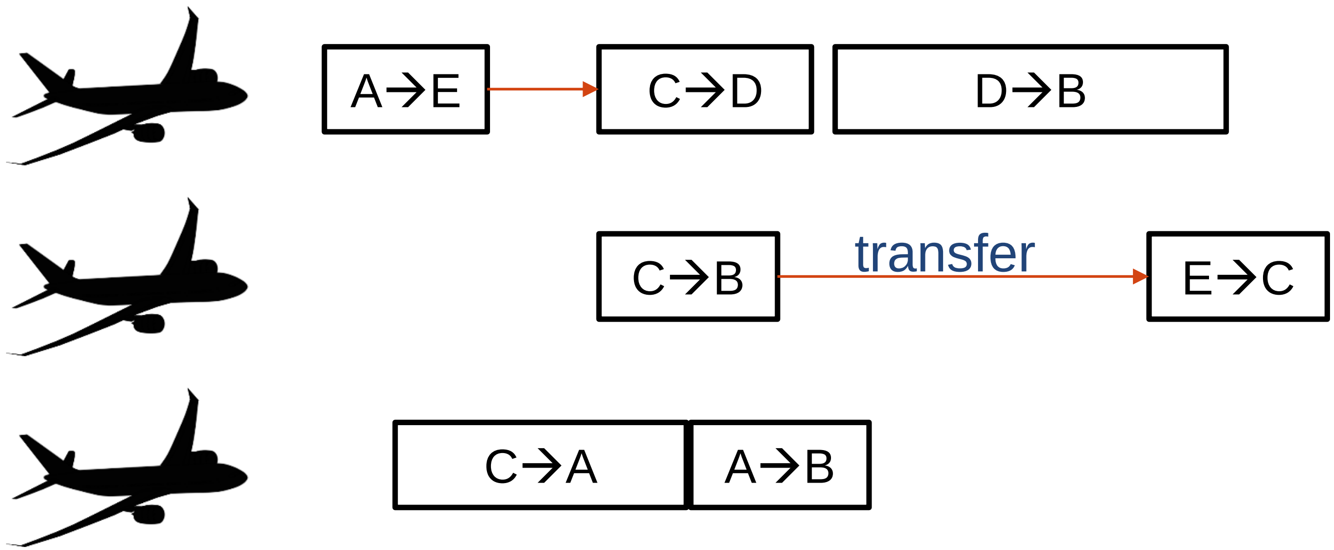
Flow-shop scheduling



Since we are looking for a permutation of jobs the model is straightforward with a single list variable

Planning

Flights to plane assignments



A solution is a partition of flights into K lists (one per plane)

The goal is to minimize the total transfer times

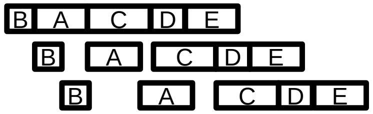
Conclusion

List Variables are a first step towards set-based modeling in LocalSolver

This higher level of modeling yields simple and compact models producing high quality solutions for

- Routing

- Scheduling



- Planning

STELLAR

- RCPSP

Leibniz Universität Hannover

- Any other sequencing problem