Overview of Stochastic Programming Applications

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1 Introduction

This document provides an overview of stochastic programming applications. We start with a simple capacity-expansion planning model that illustrates some of the key ideas of a two-stage stochastic program with recourse and the type of analyses one can perform, including risk-cost trade-offs. Next, we consider a small multi-stage financial planning model and associated analyses. These two models give a frame of reference for then discussing a variety of applications.

2 Problem Statement

In a two- or multi-stage stochastic program with recourse, it is key to clarify the timing of decisions and realizations of uncertainties. A prototypical two-stage stochastic program is a capacity allocation model under uncertain demands and/or capacity availabilities (see Figure 1). First, a strategic first-stage decision is made, allocating resources that specify the system design. Then, the decision-maker observes a realization of the demands and availabilities. Operating in what is now a deterministic setting, a second-stage operational decision is made. The first stage structure can consist of linear programming constraints or mixed-integer programming constraints. We assume the second stage structure is that of a linear program or a convex quadratic program.

This paradigm extends to the multi-stage setting in which capacity-expansion decisions are made sequentially, over time. A first stage capacity-expansion decision is made as in a two-stage model. Then, a realization of second-stage demands and availabilities is observed. Second-stage operations decisions and capacity-expansion decisions are then made knowing second-stage demands, equipment availabilities, and an updated conditional distribution on future demands and availabilities. Such a structure is also applicable to multi-stage asset allocation models in which the investment portfolio is periodically rebalanced, and we consider such an example in Section 4. While the first stage structure can consist of linear programming constraints or mixed-integer programming constraints, we assume that subsequent stages have a linear, or

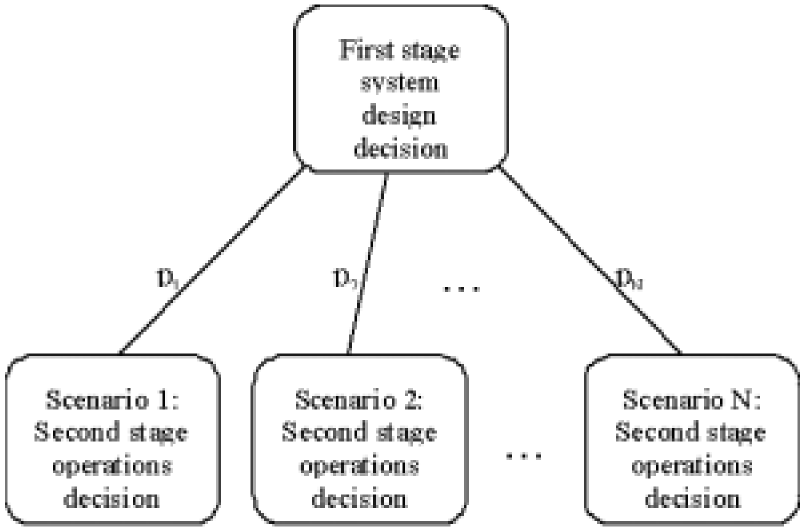

Figure 1: A common structure in a two-stage stochastic program is that the first- and second stage decisions are qualitatively different. The first stage decision designs the system, e.g., by determining capacities of various resources. This design decision must be made before knowing the exact environment in which the system will be operated. The figure depicts N such scenarios, and their probability weights, that determine this environment (e.g., different demand and reliability scenarios). The second stage operations decision is made after this uncertainty has been revealed. The goal is to find a first stage decision that is robust in that the system is well-positioned to operate over the range of possible scenarios.

convex quadratic, programming structure.

3 A Capacity Expansion Planning Example

We describe an example two-stage stochastic linear program with recourse, namely, a capacity expansion planning model in the context of power generation planning from [34]. The model contains two types of uncertainty: (i) demand for energy and (ii) generator availability. The first stage decision involves allocating generation capacity, at a specified unit cost, to each of a number of generators. In the second stage, random demands and availability fractions (rates) are revealed, and we solve a transportation problem to find a least-cost solution for satisfying demand for energy. If sufficient capacity is not available to satisfy demand then energy must be purchased, at a relatively high penalty cost, from an outside source to satisfy demand (this could also be viewed as a penalty cost for having unsatisfied demand). The goal is to find a first stage capacity allocation decision that minimizes the sum of capacity investment costs and expected operations costs.

Our instance of this model has two electric generators, three demand sectors, and per unit capacity installation costs of \$400 and \$350 at generator #1 and #2, respectively. The unit operating cost of energy sent from generator i to demand sector j is as follows:

op. costs	1	2	3
1	4.3	2.0	0.5
2	7.7	3.0	1.0

The available fractions of allocated capacity at each generator are random and have the following marginal distributions:

#1	prob.	#2	prob.
1.00	0.90	1.00	0.85
0.95	0.05	0.80	0.10
0.30	0.05	0.00	0.05

The demands in each of the three demand sectors are random and have the following distributions:

#1	prob.	#2	prob.	#3	prob.
900	0.35	900	0.35	900	0.35
1000	0.55	1000	0.55	1100	0.55
1300	0.10	1250	0.10	1400	0.10

We assume that the random parameters are independent (this is not integral to the model). The cardinality of the sample space is $3^5 = 243$. The resulting two-stage stochastic linear program can be expressed as a single deterministic equivalent linear program with 2189 decision variables and 1216 structural constraints.

We solve the problem for a range of values of the unit penalty weight for demand that is not met from our two generators. Figure 2 shows that as this unsatisfied-demand penalty grows from \$1,500 to \$26,000, we install more capacity at a higher installation cost and the probability of having unsatisfied demand shrinks. Initially, substantial decreases in the probability of unmet demand are obtained for modest increases in installation cost.

It is also interesting to compare the solution to the stochastic program with solutions obtained by other common means. First, the stochastic solution, when the penalty weight is set to \$6000, is given by:

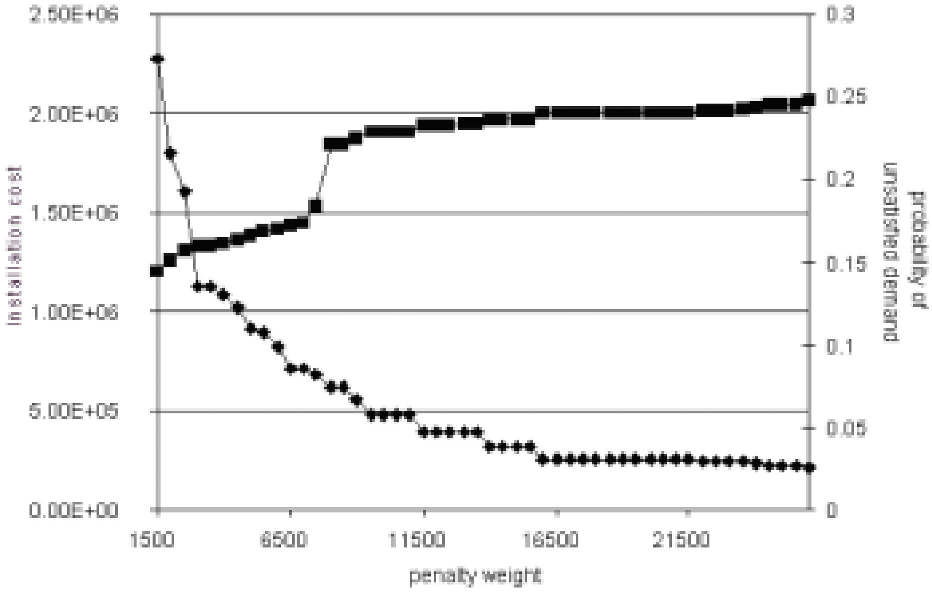

Figure 2: The figure depicts the tradeoff that a decision-maker faces in a capacity-expansion problem. As the penalty weight, depicted on the x-axis, grows the decision-maker is increasingly averse to having unmet demand. The capital cost of installation is depicted on the left-hand y-axis and grows with increasing penalty weights. The probability of having unmet demand, depicted on the right-hand y-axis, shrinks as this penalty grows.

$$x_1 = 2842.1,\ x_2 = 807.9,\ z = 1927687$$

where x_1 and x_2 denote the installed capacities and z is the sum of installation costs and expected operations costs (including penalty costs for unmet demand).

One alternative approach is to replace the random parameters with their means and solve the associated one-scenario linear program. That solution is given by:

$$x_1 = 0,\ x_2 = 3274.2,\ z = 1157659$$

It is not difficult to see how this solution arises. One unit of effective capacity from generator #1 costs $400/0.9625 = 415.58$ and one unit of effective capacity from generator #2 costs $350/0.93 = 376.34$. The higher cost of installing capacity in unit #1 cannot be offset by its cheaper operating costs and hence it is always preferable to use only generator #2. The total installed capacity is $(995 + 990 + 1060)/0.93 = 3274.2$. Note the objective function value of this so-called *expected value problem* is less than that of the stochastic model (as is predicted by Jensen's inequality).

The expected cost of the expected-value solution in the stochastic environment is 2433952, which is 26% higher than the stochastic solution. The probability that the expected-value solution will have unserved demand is 0.30 (contrast with 0.098 for the stochastic solution, with the penalty weight still at 6000).

Another common strategy is called *scenario analysis* and obtains a collection of first stage decisions $\{x^\omega\}$ by solving $|\Omega| = 243$ separate linear programs (one for each demand/availability forecast), and then combines these first stage decisions in some "reasonable" way to obtain a solution x. The combination formula we will use is $x = \sum_{\omega \in \Omega} p^\omega x^\omega$. This technique yields the following solution:

$$\boxed{x_1 = 459.7,\ x_2 = 2607.3,\ z = 1108915}$$

Note that the objective function value is smaller than that of the stochastic solution; this is as expected since the scenario analysis value corresponds to making decisions under perfect information regarding the future. The expected cost of the scenario analysis solution in the stochastic environment is 2629970, which is 36% higher than the stochastic solution. The probability that the scenario analysis solution will have unserved demand is 0.53 (again, contrast with 0.098 for the stochastic solution).

4 A Multi-Stage Asset Allocation Model

We now turn to a multi-stage recourse model for asset allocation [3]. Like the capacity-expansion planning model of the previous section, this is a small "toy" model but is valuable for gaining insight into the kinds of solutions stochastic programs can produce. In this model there are two types of investments: *stocks* and *bonds*. The returns on these investments are assumed to be random variables with known distribution. The purpose of the investment is to provide for a child's college education Y years from now. We begin with an initial wealth W and after Y years we would like to exceed the target tuition goal, G. We assume that the portfolio can be restructured every y years so that there are $T = Y/y$ investment periods.

Each time period the market is either "high" or "low." If the market is high, it would have been preferable to invest in stocks, and if the market is low, it would have been preferable to be in bonds. We consider a model over 15 years in which the portfolio can be rebalanced every five years. This leads to a four-stage model: three re-balancing decisions are made at the beginning of years 1, 6, and 11, and at the end of year 15, we simply have a cash-out decision that computes the value of our

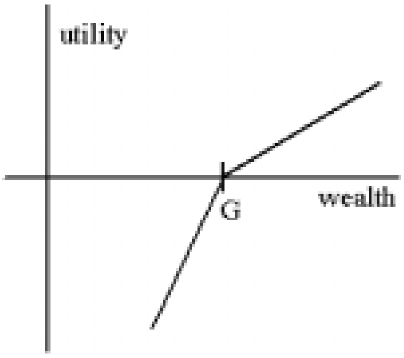

Figure 3: Utility function for wealth above and below target G. As the ratio of the slope of the left-hand piece to that of the right-hand piece increases, the decision-maker is increasingly averse to having returns below the target.

portfolio relative to the target. The model has a total of 8 scenarios corresponding to the market being high-high-high, high-high-low,...,low-low-low over the three five-year periods. The concave utility function of Figure 3 represents an aversion to risk. We denote the ratio of the slopes of the two linear pieces in Figure 3 as ρ. As ρ increases the decision-maker is increasingly risk averse to failing to meet the target wealth level of G. (One possible interpretation of the ratio of slopes of the linear pieces of the utility function has to do with the ratio of the cost of borrowing to the value of future income.)

The wealth at the end of the 15-year planning period is a random variable that takes up to eight values corresponding to the eight possible scenarios. Figure 4 shows the distributions of wealth for three different values of ρ. When $\rho = 1$, the distribution is symmetric, but as ρ increases larger up-side returns are sacrificed in order to pull the left-hand tail of the distribution closer to the target.

5 Application Areas

In this section we point to a number of different areas in which stochastic programming has been applied.

- **Finance:** Applications in finance include security pricing, asset-allocation, and asset-liability management. This is one of the most popular applications of stochastic programming today. A multi-stage asset-liability management model was developed by the Frank Russell Company and the Yasuda Fire and Marine Insurance company [6, 7, 8]. They report outperforming a fixed-mixed strategy (the previous strategy) by $79 million in income return and $9 million in total

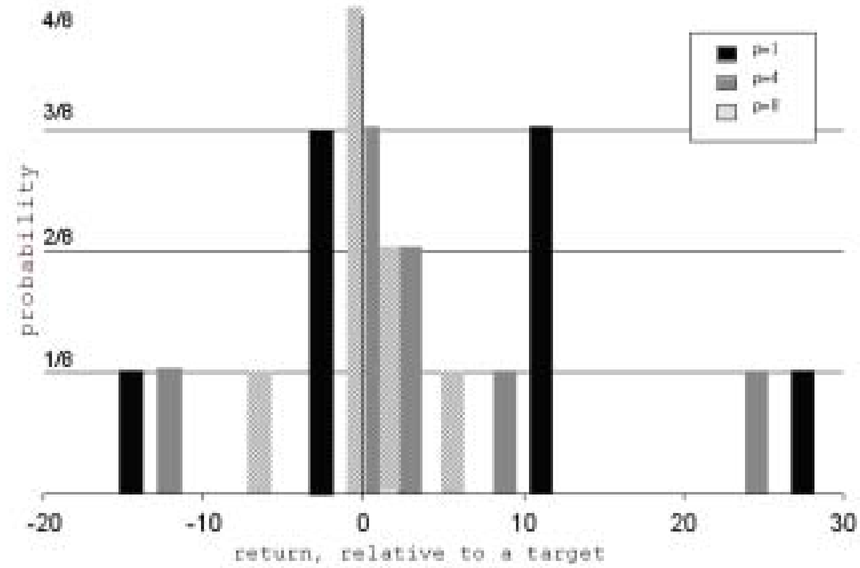

Figure 4: The figure depicts the distribution of returns on investment, relative to a target, with three different levels of risk aversion. Negative returns denote failure to reach the target and positive returns are excess values relative to the target. When $\rho = 1$ the distribution of returns is symmetric. As ρ increases to 4 and then 8, risk aversion to failing to meet the target increases and the distribution of returns tightens about the target.

return over two years. Towers Perrin-Tillinghast use a stochastic program for asset-liability management for pension plan and insurance clients [50, 51]. They report saving $450-1,000 million in opportunity costs in US West's pension plan. (The work reported in [51] was a finalist for the 1999 Edelman Award for Management Science Achievement sponsored by INFORMS.) ORTEC Consultants have implemented a multi-stage stochastic programming for pension planning in the Netherlands [5]. Stochastic programming models have also been used at Fannie Mae [29, 30, 31]. Computational results for a class of 10-stage asset-liability management models with up to 2688 scenarios are reported in [11]. A group at Deutsche Bank uses stochastic optimization models for asset allocation in hedge funds [15, 62]. For an overview of asset-liability management systems see, e.g., [81].

- **Electric Power Systems:** A multi-stage stochastic linear program for hydro-electric scheduling was developed at Pacific Gas & Electric Company [35]. Their largest problems (on a single hydrological basin) consisted of about 70,000 constraints and 250,000 variables and solved in about six minutes on an ($\approx$ 1991 era) HP 9000/750 workstation. A similar multi-stage model for hydro-electric scheduling in a Brazilian system is reported in [59, 60]. Unit commitment models are typically difficult discrete-optimization problems, but there has been some work on stochastic unit commitment of thermal, and to a lesser extent hydro, systems [25, 56, 61, 70, 71]. Examples of capacity planning models in the electric power industry include [13, 47, 54]. There is growing interest in models combining finance and electric power systems and some of this work is via stochastic programming [19].

- **Production Capacity Planning:** The work of [37] with Jeppesen was the winner of the 2000 Edelman Award for Management Science Achievement. Part of their work involves solving, via a heuristic, a multi-stage stochastic program for evaluating the benefits of flexibility in production equipment. The stochastic programming work is reported in greater detail in [36]. They report annual savings of $1.4 million, and they estimate that traditional approaches under-valued production resource flexibility by 26-54%. A production capacity planning model for General Motors is described in [16]. Their model is a two-stage stochastic mixed-integer program with integer variables in the first stage. The authors report that the model provided analytical support for closing two-four plants and motivated GM to carry out further analyses on products that were not included in the recommended product mix. A related stochastic programming model for capacity planning in semiconductor manufacturing was employed at IBM [32]. Other production planning work for capacity decisions includes [9, 18, 26, 67, 69].

  Two-stage stochastic programs for deciding production levels (as opposed to capacity expansion planning) in the face of uncertain demand include [10, 33]. A two-stage model for manufacturing, assembly and distribution motivated by the automotive industry is presented in [17].

- **Communications Networks:** A capacity expansion planning problem on a high-bandwidth private-line network is considered in [66]. A simulation model built at Bellcore provides estimates of system performance for a specified network design. A two-stage stochastic program replaces the simulation with a mathematical programming model. This approximation is validated by testing candidate network designs in the simulation model. For further work in telecommunications networks, see [21, 68, 73].

- **Managing Natural Resources:** There is a significant volume of work in managing water resources, e.g., [20, 40, 44, 46, 74, 76, 77]. The model in [77] is a two-stage stochastic program in which first stage decisions locate pumping wells in the face of uncertainty governing the underlying hydrological conductivity field. The second-stage problem is a convex quadratic program in which pumping rates are determined in an attempt to contain a contaminant plume and environmental constraint violations are penalized. The second-stage constraints arise from a finite-difference approximation of the differential equation governing groundwater flow. Other examples of managing natural resources via stochastic programming models include fisheries management [27], forest harvest management [22], and policy models regarding greenhouse gas emissions [4].

- **Military and National Security Applications:** Early research in interdiction models was motivated by military applications [23, 48] and concerned deterministic mathematical programs to disrupt flow of enemy troops and materiel. Similarly-motivated work continues, e.g., [78]. Some of the interdiction work in the 1990s, e.g., [75, 79], was motivated by the desire to reduce flow of illegal drugs and precursor chemicals into the US. In [12] the work of [79] on interdicting a maximum-flow network is generalized to allow for random arc capacities and random interdiction successes. The Second Line of Defense (SLD) program is a collaboration between the US Department of Energy and the Russian Federation State Customs Committee to help strengthen the overall capability to prevent the illicit trafficking of nuclear material, equipment, and technology across Russia's borders (see, e.g., [2]). The SLD program has prompted recent research in stochastic network interdiction [49, 57, 58]. There are also stochastic programming models for military logistics. The deterministic airlift model of [1, 64] has been extended by [24] and [55] to incorporate random groundtimes due to aircraft reliability. Another stochastic programming model for airlift is developed in [52] and [53].

- **Vehicle Fleet Management and Location Problems:** Discrete stochastic facility location models select facility locations, and possibly their capacities, from a prespecified candidate list and then satisfy customer demands from the selected sites. The models that have been studied differ with respect to the timing of decisions (facility location and capacity, assignment of customers to facilities, and delivery) and observations of the random demand. See [45] for a survey. Three different classes of models have the observation of demand after the delivery decision is made, after the assignment of customers-to-facilities decision is made, and after the location-capacity decision is made, respectively. demand and then penalties are paid for shortages location and sizing decisions and the assignment and then the second stage decisions simply assignment of customers to facilities until the and capacities of the facilities are fixed, Combinatorial stochastic routing problems are stochastic variants of the classical deterministic (multiple) traveling salesman problem and capacitated vehicle routing problems. Computationally tractable models here do not have recourse. Instead they usually penalize up-side deviations from target completion times or failure to satisfy demand. Laporte and Louveaux and their colleagues (many of whom are at Centre de Recherche sur les Transports in Montreal) are among the leading academic researchers in these two areas; see, e.g., [41, 42, 43]. Stochastic programs with continuous decision-variables for vehicle routing include stochastic variants of min-cost flow vehicle allocation formulations. They differ from the combinatorial models in that vehicles have the opportunity, but not the obligation, to satisfy demand requests. Powell [63] and other work motivated by his models [14] are in this vein. Some of these models are multi-stage problems (say, up to 5-6 stages) in which demands are revealed sequentially over time. Some of the work of Powell has industrial support and use (see `http://www.castlelab.princeton.edu`), although only a subset of his work involves stochastic programming, per se. For a survey of stochastic location, routing and allocation models see [39].

- **Airlines Industry:** Stochastic programming applications motivated by airlines industry problems have been limited, to date. That said, the growing body of work in "recovery" models (e.g., [72]) suggests there is a need for robust scheduling. The work of [65] and [80] concerns airline crew scheduling. Other related stochastic programming work relates to seat inventory control and pricing [28].

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